Mathematics and Beauty
November 14, 2009
2:30 PM
The Philoctetes Center

Levy: Francis Levy
Nersessian: Edward Nersessian
Brann: Eva Brann
Greene: Brian Greene
Livio: Mario Livio
Mazur: Barry Mazur
Scarry: Elaine Scarry
Audience: Question from audience

Levy: I'm Francis Levy. Ed Nersessian and I are co-directors of the Philoctetes Center, and welcome to *Mathematics and Beauty*, the greatly awaited second roundtable in our series on mathematics and imagination.

Before we go on I wanted to call your attention to the exhibit on the wall. A lot of people walk into Philoctetes and they think that the art is decorative, and they look at it as wall covering. It is not. All the shows that we do, which are curated by Hallie Cohen, in tandem with Adam Ludwig on our staff, are related to roundtables, and this is on aesthetics and mathematics.

Now I'm very happy to introduce one of our great Philoctetes supporters, Barry Mazur. Barry Mazur is Gerhard Gade University Professor at Harvard, where he teaches in the Mathematics Department. He is the author of *Imagining Numbers (Particularly the Square Root of Minus Fifteen)* and his research interests include number theory, automorphic forms, and related issues in algebraic geometry. He is the winner of the Veblen Prize, the Cole Prize, and the Steele Prize from the American Mathematical Society, and has been elected a member of both the National Academy of Sciences and the American Philosophical Society. Barry Mazur will moderate this afternoon's roundtable and introduce our other distinguished guests.

Take it away, Barry.

Mazur: Thanks loads, Francis. I am really honored to be in a panel with our other guests. Starting on my left is Eva Brann. She's taught in St. John's College for over half a century and has written a number of wonderful books, including *The World of the Imagination: Sum and Substance*. Her interests are enormously broad, and every time I ask a question about beauty I ask it first of her—and then of Elaine, who is Professor of English at Harvard University. She's a colleague of mine—in fact, you're William M. Cabot Professor of Aesthetics and the General Theory of Value at Harvard, and has written some magnificent books, including *Dreaming by the Book*, and *On Beauty and Being Just*.

To my left is Brian Greene, who I think must be known to everyone in the room for his wonderful expository work on bringing string theory to comprehension to people who are interested but not necessarily technically in the field. Now, that's a very difficult thing to do, and *The Elegant Universe* and *The Fabric of the Cosmos* is sort of a great contribution to our literature, as is the work of Mario Livio, whose book *The Golden Ratio* is an absolutely wonderful contribution as well. Mario Livio is a senior astrophysicist and Head of the Office of Public Outreach at the Space Telescope Science Institute. Brian Greene is co-director of Columbia University's Institute for Strings, Cosmology, and Astroparticle Physics.

I think I was the designated moderator, and that means I was meant to introduce the people of the roundtable to you. I also introduced the roundtable to you as the first example of an element of beauty in mathematics. Eva, what do you think about that?

Brann: I was thinking about what might have been meant by saying that Euclid alone has looked on beauty bare, which is probably the funniest line of poetry ever written.

Levy: What is it again? I didn't get that.

Brann: Euclid alone has looked on beauty bare. That's Edna St. Vincent Millay, and if you think of it properly you have to laugh. And I wondered what an example might be of bare beauty. That doesn't mean beauty nude, it means beauty in some pure way. And certainly the ancient tradition is that circles are the most beautiful objects in the world, so I began to think about what makes them beautiful. I'll only give the last one of my conclusions, that one of the beauties of a circle is that if you sit around it everybody's equal, and King Arthur's Round Table was established for that purpose, so that all the knights should be the same. So that is one of the beauties of a circle; every place on it is, as is often said in the literature, beginning and end at once. Also, it has a determinable center, just one, and there are dozens and dozens of other good characteristics that it has.

But, I want to finish this speech by saying that when I put all those things together I didn't know what about it exactly was beautiful. It seems interesting, but is interesting the same as beautiful? So I end with a question.

Mazur: This will be my last moderational chore. Eva ended with a question, and the minute people have questions I hope it's okay, Ed and Francis, if the minute anyone has a question, jump in—

Levy: One thing Barry—

Mazur: It's not okay.

Levy: We try to let the roundtablists talk for a while first, and then we have a period of time, leaving at least a half an hour, forty five minutes—

Mazur: I see. All right. So, I think the other moderational duty is to sort of ask other people to chime in with questions—I do have a question as well, but I'll leave that for later.

Nersessian: It seems to me Eva said it's interesting but is it beautiful, but what is beautiful? You know, what is the definition of beautiful? That's your question.

Scarry: Well, I agree with Eva's description, and I know that not only the circle but the sphere was beloved—

Brann: Even more.

Scarry: Yes, even more, by Parmenides and Plato and Boethius as the most perfect of forms because every point on it was equidistant from the center.

Brann: Yes.

Scarry: And I think that when they invoke the sphere they're not always explicitly thinking of the principle of equality the way we do—that is, political equality—and yet it certainly is anticipatory of it. And so even something like, say Augustine's *De Musica*, where he talks about the way in which we may first come to know numbers through the bodily experience of rhythm, he talks about equal measure and equality as the most perfect event, and he sees it in musical form and he sees it in the smoothness of surfaces, and even in the uniformity of the color of the sky, and certainly he sees it as an attribute of God. And again, I don't think he's talking about political equality, but I think that it is making audible or available principles that in philosophers—philosophers who do care about equality as a principle of justice—we hear again, as in Rawls's famous definition of justice as fairness requiring asymmetry in all our relations with one another, or in the many centuries of talk about the necessity of an equal balance between, say crimes and punishments or work and compensation and so forth.

Livio: I guess, you know, when I came here and I thought about this math and beauty—you mentioned the word mathematics and beauty, and I thought that there are actually a few rather different things which are related to this topic. One would be of the type of when do mathematicians call something beautiful. And there is a whole series of things that have to do with that, and we can maybe speak about them later, but that would be one thing.

Then there is a second thing, which relates to the question you asked, which is, you know, even every person on the street has a certain sense of aesthetics and of beauty—by the way, artists don't like so much the word beautiful. They use other words, but there are some things that we call aesthetic and so on, and the question is, is mathematics in any way related to the concept that the ordinary person would call beautiful?

And just as a tiny example, I mean the two of you just mentioned the concept of symmetry, which relates to the circle, that relates to many other things and enters into this. And some people try to carry it to extremes, like, you know, a mathematician that I'm sure you know very well, [laughs] and probably you do, G.D. Birkhoff even tried to develop a mathematical theory of beauty. I don't think it's a very good theory, but he nevertheless tried to do that. So that is a second type of question that relates math and beauty.

Then there is another question—you know, I am an astrophysicist, so what I try to do is explain the universe. When do we call a theory of the universe beautiful? What are the requirements for that? And there is a whole series of questions that are related to that.

And finally, there is something which I once dubbed passive effectiveness of mathematics, namely there is a sense when mathematicians do something with absolutely no application whatsoever in mind, and somehow decades later, or sometimes centuries later, that very precise branch of mathematics turned out to be a very precise explanation to some natural phenomenon, and they see great beauty in that.

So there are all these different things, and after I let everybody speak I'm happy to give examples from each one of these, but there are many relations between the word beauty and the word mathematics.

Greene: Just to add maybe one thought to that, I think the idea of judging whether something is or is not going to be characterized as meeting a criterion of beauty, I think oftentimes one views it as a static type of process. You look at something and you try to assess it on some scale or another—and as you're saying, there are many different scales and many different domains. But I think there's another side of it which is more dynamic, which is, in particular, as you're saying, when you are a physicist, as we are, trying to use mathematics to actually describe something, there is a sense in the actual work, there are these rare moments when the mathematics seem to be driving you, as opposed to you driving the mathematics. And there's a sense at that moment of an alignment between this abstract body of understanding and knowledge with this real world, this real universe that you're trying to use it to describe, and it's that close alignment, step by step by step, driving you forward, that gives you some inner sense that you're touching something spectacular, something beautiful, something that is really—

Livio: Right, almost an inevitability.

Greene: That's right, exactly. I think that actually is the word that I would use too, this sense of you not making choices. The mathematics is making the choices for you, and it's a sense, as you say, of inevitable motion forward in an effort to describe something real, as opposed to something abstract.

Mazur: Since I have two physicists here, let me say that it goes the other way as well—I mean very much so, and especially with modern theoretical physics, and string theory in particular, where the string theorists for the past twenty years have been coming to mathematicians and telling them, first that their view of the subject—the mathematician's view of the subject, their own subject, mathematics, or a specific aspect of mathematics—is not broad enough, and making incredibly precise predictions about what will happen if one expands one's viewpoint. Incredibly precise to the extent that there are specific enumerations, which are of great interest to the mathematicians, and yet the mathematicians didn't feel that it was the right time even to pose the questions. The physicists come and say to them, "Because of our physical intuition we feel that there are twenty-two million fifth degree genus zero curves on a three-dimensional fifth degree space," and the mathematicians will very often be able to prove it, but they will never have the intuition, or at least they haven't so far achieved the intuition that seems to come naturally to the physicists, that allowed the physicists to make this prediction.

Greene: Well, in fact the mathematicians actually said the prediction was wrong.

Mazur: The mathematicians said the prediction was wrong. Exactly. I didn't want to say that if there are any mathematicians in the audience. But, yes, the physicists with their intuition somehow predicted something that corrected—and it's a very large number, by the way; it was not correcting a small numerical error—corrected something very precise, but that one thing was one of many, many, many things that had happened over the past twenty years.

What is incredibly beautiful in mathematics that's hard to pin down is intuition and sources of intuition. So if you think of it, when we talk about beauty, if you look around the room, we're seeing visual beauty. And one of our great mathematical intuitions is being able to envision things. The physicists have other intuitions, which don't have words, at least in my vocabulary, and yet produce these sort of wonderful, let's say directives, directives for further understanding. And that itself, if we have a forum for math and beauty, it seems to me that we should also celebrate the fact that part of the beauty of mathematics is the fact that there are these marvelous intuitions that we, all of us have, and we're constantly improving or sort of making more, let's say making sharper, making more imaginative. That itself is an object of beauty.

So, although we should celebrate the sphere and the circle, I think we should celebrate the increase of intuitions that comes along with the interplay of mathematics and our thoughts, and particularly the interplay of physics and mathematics and our thoughts.

Greene: Since you go in that direction, let me just elaborate one point on that, because we did start with your suggestion, talking about symmetry, the circle and the sphere. And in terms of the way a physicist's intuition may differ from what you might expect based upon these kinds of ideas and these kinds of objects, what we find in physics is that in many ways the most beautiful structures are the ones that are almost but not quite symmetric, and that isn't something which is particularly unfamiliar. You know, we love asymmetry in a way that we love symmetry too, but in our efforts to use math to describe physical phenomena, we found that you have to move away from the symmetric settings. We have a very specific name for it; it's called spontaneous symmetry breaking, and it's a phenomenon familiar to most physicists, but it's basically a mathematical description of an almost symmetric situation. And what we find is that the universe seems to be almost symmetric in many, many ways, and it's almost symmetric this way and that way, and based upon those almost-symmetries you can make predictions about what particle you should see in a collision at an accelerator. And to me it's the most astounding thing that you use this almost symmetric mathematical description and it tells you there should be a particle that weighs 178 times that much of a proton, or 93 times that of a proton, and you go to these accelerators, you smash particles together, and goodness gracious, there's just that kind of a particle in the debris from the collisions. *That* is beautiful.

Levy: Brian, that's an interesting point because Mario Livio brought up a point of what most people think of as beauty, beauty in contravention to what scientists might consider, or mathematicians, but your definition sounds a little bit more like an aesthetic definition of beauty, which is playing between order and disorder, or what we do in aesthetic concerns—

Brann: Yes, I was going to say, it aligns it with the way that one thinks of beauty in non-mathematical, non-physical cases, which is, one might say, flawed perfection.

Levy: Yes. Yes.

Livio: But let me nevertheless say that, I mean, Brian tried to emphasize this slight breaking of symmetry, but we wouldn't be thinking about this slight breaking of symmetry, or spontaneous symmetry breakdown, if there wasn't the symmetry, you know, that you speak about. So this slight breaking of symmetry, it allows you to move and to develop things that are complex. For example, you know, he mentioned these particles. If there was a precise symmetry between all the particles and anti-particles in the universe, then we wouldn't be here to talk about this, because all these particles and anti-particles would annihilate each other, and the whole universe would just be filled with radiation and we wouldn't be here. Because of this tiny, tiny break of that symmetry, which is like one part in three billion in the breaking of symmetry between particles and anti-particles, this is what allowed us—well, galaxies, stars, us, everything else to emerge.

So, there is an incredible symmetry that's underlying that, but then you need this tiny break—you know, it's like Cindy Crawford's mole. I mean she's perfectly symmetric, but then she has that thing which makes her a little bit more interesting.

Mazur: I think that pins it down, doesn't it?

Scarry: Well, I think it does pin it down, but it also is important to keep your overall point in mind, that without the large fact of symmetry even the asymmetry wouldn't be interesting. Sometimes my students will say to me, "Well, isn't a face"—like the one you mentioned—"actually more beautiful because of the asymmetry?" But we're talking about a face that's 99.99% symmetrical, and if any of us were suddenly to have an injury—let's say if we were in Iraq or had some terrible cancer or something of the bone—that would not be perceived as beautiful. And I often think that injury is operating as one of the opposites to beauty. So I think it is important to underscore something that both of you have emphasized, the tiny break in the symmetry.

I think that one of the things that's really interesting about what you introduced, Brian, is that one of the puzzles to me about math and beauty is the question of where it starts. That is, when you're talking about non-mathematical context, beauty is often at the threshold of the work that you do. So, for example, the famous description in Plato's *Phaedrus* of what happens when you come into the presence—the example he gives is you come into the presence, Socrates comes into the presence of a beautiful boy, and suddenly, you know, his wings are beginning to sprout and he's breaking into a cold sweat, because he's remembering the immortal world, where things like truth and goodness and justice reside. And this is now the beginning of a call to work on those things, so beauty is really instigating acts of education and searches for justice, and throughout the whole tradition we've had versions of that, of the description of beauty as a call, or as a greeting to some new endeavor—

Brann: Elaine, if I may carry on where you started—

Scarry: Yes.

Brann: It's in *The Phaedrus*, Plato's most romantic dialog—

Scarry: Yes.

Brann: —or, second most romantic dialog, that there is a one-line definition of beauty which strikes me as really right. He says beauty is visibility.

Scarry: Yes.

Brann: That's what it means to be beautiful, to be seeable.

Scarry: Yes.

Brann: How does that strike people?

Scarry: If something's beautiful, then you can see it—

Mazur: It might be the key to why there's any relationship *at all* between math and beauty.

Brann: Yes.

Mazur: In fact, if you think of it, it's pretty strange. I mean mathematics has a primary mission to explain things. Now, if you have to go into the muck and deal with the ugly truth, you would do it, because your mission is to explain, not to produce beauty. And yet no matter what type of mathematics you do or what type of mathematics you learn, you read, you think about, it's suffused in beauty. And the question is—the question for me is, has always been, why? I've never really understood it. At one point I thought of it as a bonus, as if for hard work. You prove a theorem and you're a hero. Your payback is it's beautiful. If it weren't beautiful you'd be happy to prove it also, but it's a bonus.

But linking it with visibility sort of makes a much firmer link in some sense between seeing things, between epistemology if you wish, and beauty. And maybe that's why not only is the ideas of mathematics beautiful, but also everything that's in this room, that surrounds us, which are mathematical structures, if you want, beautiful as well.

Brann: I think he means not only that it is in itself visible, but that it gets to us as visible. You know, there's both aspects of visibility.

Livio: There is certainly an element of this which has a history which, maybe I'll just tell a small story: There was this Italian mathematician, Giralamo Cardano. He lived in the sixteenth century, and he wrote this famous book about mathematics, *Ars Magna*. And he solves there equations of first degree, second degree, third degree, and actually with the help of a student, Lodovico Ferrari, also fourth degree. But in the introduction—you see, people at the time thought of a linear equation that describes a line, and a quadratic describes an area, and the cubic describes a volume. So in his introduction he says that he will show the solutions to all of those equations, and he will in passing discuss also higher levels, which he meant the quartic, the fourth power. He says, "But only in passing, because nature does not allow such things."

And then there was another mathematician in the seventeenth century, John Wallis, in whose books Newton actually studied mathematics, and Wallis said if you run a line into a line you get a plane, a line into a plane you get a solid. But what happens if you run a solid into a solid? Is this a planar plane or what? "This is a monster of nature," he says.

So at that time what was not really visible, if you like, to them they regarded as a monstrosity. After that, you know, Lagrange started a little bit and so on, eventually of course Einstein and so on. Fourth dimension is by now, children in elementary school already learn about it. And if you work in string theory you talk about ten, ten plus one, maybe twenty six dimensions and so on; all of these things are regarded as beautiful now, because they are, if you like, visible. I mean they're not quite visible, but they're visible at least in our mind's eye.

Greene: But is visible the operative word—in other words, something that I've struggled with for a long time, which goes along exactly the same line of thinking but perhaps comes to a different outcome, is as our physical theories evolve to explain different data that we get—so we get data from the cosmic microwave background radiation, we get data from accelerator experiments. We find we have to modify our theories to describe the data. Does our aesthetic sense evolve along with the changing theories that are driven by data, so that no matter what our theory ultimately is we'll say, "Oh, that's beautiful," and it's beautiful because it describes the data correctly, or is there some innate, immutable aesthetic sense that will ultimately be brought to bear, and even if theory describes, the universe will say, "No, that doesn't feel beautiful. That's not going to meet"—

Livio: I think, and you will tell me if you agree with me or not, I mean I think that there are some elements that we all like to be there. So, one is reductionism. We would like to have a theory that is as simple as possible that describes as many things as possible. It's almost like, you know, when I was in elementary school they taught us that in western tradition one God is better than many gods. And I remember that even then I wondered about this a little bit. No, because if it's a god, you know, what do I care? There can be a god for every phenomenon. But, you know, still we had this feeling that one God somehow unifies everything. So we try to do that, to have this thing.

So, you take Einstein's general relativity and you try to expose this to the general public and so on. I think that—there is no question that the equations of general relativity are more complicated than Newton's equations. But somehow I feel, and I believe you feel, that actually the underlying principle of general relativity is simpler, because in Newton's you have this sort of mysterious force that acts at a distance and it's not clear what it means and so on, and in general relativity suddenly it is really all the structure of space that does the whole thing.

Mazur: Are you suggesting that simpler is an aesthetic issue here?

Livio: Yes, I believe that at least in physics—I'm trying to answer Brian's question—

Greene: But that's part of my point. It's not obvious to me that Einstein is simpler than Newton, and I think that there are many people who wouldn't at first sight come to that conclusion. I do think that after people study differential geometry and study the equations of Einstein and see how they're able to describe so much of the world from a very economical starting point they'll perhaps agree with you, but that to me may be the evolving aesthetic sense that I'm talking about—

Livio: I think so—

Greene: As opposed to—you know, if I wrote down the equations of Newton on this board and the equations of Einstein on that board, say for this group, I don't know how many people are technically trained, but my guess is that people who are not technically trained by and large would think that Newton is a hell of a lot simpler than Einstein.

Livio: I agree.

Levy: So wasn't the common denominator in Keats, go back to Keats. Isn't visibility equal to truth? And beauty coming, emerging from the discovery of—

Greene: Maybe so, if by definition visibility is, in this limited context, the ability to actually describe what's really out there.

Mazur: I think the evolution of the aesthetic, corresponding to our understanding, is shown everywhere in the history of mathematics. In the sixteenth century people hated negative numbers. Cardano, when he had a cubic equation—there's not a single negative number in his *Ars Magna*. If he had a negative number he would put it on the opposite side of the equation to make it positive.

Livio: And the imaginary numbers he called sophistic.

Mazur: And the imaginary numbers he said, when he had to multiply two imaginary numbers he told his readers, be prepared, "Dismissing mental tortures, multiply these things."

So, at that point there was a certain amount of mental, let's say hardship, and this hardship is surely shown in kind of an aversion, an aesthetic aversion for certain aspects of mathematics.

Brann: I have to tell you that it persists into the twentieth century. I had a colleague who hated negative numbers. It turned out her divorced husband was a number theorist. [laughter] But she hated them. One might.

Mazur: Okay. Yes, you're good. But I guess my idea was that the minute you understood them you loved them. And loved them not only in terms of passion, but in terms of aesthetic appreciation. But maybe I'm wrong.

Brann: I want to say, I came here hell-bent on getting a mathematician or a physicist to tell me what seems to be necessary to know in order to talk about beauty in either of these fields, namely what ugliness in them would be. What would be an ugly piece of physics or mathematics?

Livio: Fine-tuning. If you have a physical theory that requires a lot of fine-tuning, I don't know of a single physicist who will think that's beautiful.

Mazur: I should say, we were supplied with a list of quotations from our hosts, which are apt. And one of them from the mathematician Hardy talks somewhat to that, and I think gives the answer that probably every working mathematician or physicist would agree to at least. He says, about a theory, "Beauty is the first test. There's no permanent place in the world for ugly mathematics." So what ugly mathematics really means, at least for Hardy is unfinished business. That is to say that when you see something that's ugly you sort of train yourself, or you get the instinct somehow that you don't leave it. There's something you do not understand.

So, ugliness is actually perhaps more interesting than beauty, because it leads you to things that you don't understand. I mean that seems to be the moral that you would get from Hardy's quotation.

I think you have a similar view—

Scarry: I think I do, because—sometimes people have complained that in the little bit of writing I've done on beauty I don't ever use the word ugly, and it's because it never rings true to me as an opposite for beauty—although what does make a lot of sense is descriptions such as unfinished business. And, you know, going back to I think Brian's earlier description of knowing you're on the path to something and sensing that there's something there before you're even there, and so that the state before that is some kind of deficit state, or the falling away from it is a deficit state, and that's why I alluded earlier to injury. We don't want to say, oh, well if you're injured you're outside the realm of the aesthetic, but I think that we have a deep aversion to injury in the sense that we want to avoid inflicting it, and if we see it we want to repair it, that is, we take that to be unfinished business. And if we can't change the injury—let's say if I can't walk we want to repair the injury so I can walk, and if we can't repair the injury then we want to revise the world, through, let's say ramps or lifts and buses and so forth, so that that no longer exists as an injury.

But I think that both those ways of thinking of it, as unfinished business or something that requires our address, are accurate ways of understanding what might more—I want to say more clumsily be referred to as the word ugliness, because I don't think that word actually is, for me anyway, very useful.

Mazur: Okay. Can I ask you a question about—a marvelous question you posed, to yourself and then to other people, in one of your essays, *On Beauty and Being Wrong*, at one point you discovered that your hatred of palm trees was wrong. And so there's kind of an aesthetic reversal there.

Scarry: Right.

Mazur: And, by the way, the rest of that essay is filled with palm trees. So you changed. You changed.

Scarry: Right.

Mazur: And it occurred to me just as you were speaking that we might ask everybody, are there moments sort of in conceptual realms—ideas, rather than sort of physical objects—where there's something that one hated, and then all of the sudden there's this reversal and you say, "I love it." For example, the person who hates negative numbers. It might happen at one point that he will have an "aha" and love it.

Now, has this ever happened—I mean there's a number of times it's happened to me, but has it ever happened to any of us, or in the audience, and it might say something about what we really think beautiful is in a conceptual realm if we focus on a personal moment of reversal.

Brann: Will the opposite do?

Mazur: Uh-oh.

Brann: I'm sorry to say Elaine, although I read your book and appreciated it, I used to think palm trees were rather beautiful. When I was in Miami recently I thought to myself they're just large dish mops.

Mazur: Okay, okay, you fight it out.

Scarry: I think the point I was making is that, talking now about the non-mathematical world, that I think the way—you know, people often say this is the idea that was alluded to earlier of Keats, "Beauty is truth, truth beauty," is that really true? It seems to be true in math, at least to a layperson outside of math. Whereas in most of the world it seems as though it has, beauty has a relation to truth but isn't necessarily identical with it. And that's what I was trying to get at with the description of error, that if you—most people will remember some time when they've really changed their mind about something beautiful, in either direction. Either somebody or something they thought was beautiful now—you know, Emily Dickinson says, "It dropped so low in my regard I heard it hit the ground." Or the opposite, that they hadn't recognized something and they did. And I felt that this had something to do with the way in which beauty addresses the plasticity of the mind and the kind of, our ability to kind of carry out very limber mental acts, and so I was actually interested by the description that over time there are changes in mathematics, because the sort of layperson's view of mathematics is that once a proof is there it never changes, and once certain ideas about math are there they don't get altered. And I feel as though I'm hearing today that—

Mazur: Both from Mario and Brian—

Scarry: Yes, exactly.

Mazur: —and me, that it's a moving target.

Scarry: Yes, a moving target. Yes.

Mazur: And it somehow tracks something about comprehension or—yes?

Livio: In mathematics I think it's slightly more complicated than that in the following sense—I'll use an example that is a well known example to mathematicians, but for centuries Euclidian geometry—that's the geometry we learn in school—was regarded not just as a branch of mathematics. Euclidian geometry was space. Space was Euclidian geometry. People like Immanuel Kant said there is no way to describe space other than Euclidian geometry.

And then in the nineteenth century something shocking happened. Three mathematicians independently—Carl Friedrich Gauss in Germany and Janos Bolyai in Hungary and Nikolay Lobachevsky in Russia—showed that they can drop one of the basic axioms of Euclidian geometry, and they can construct new types of geometry that look very different—I mean you can construct them on a sphere or you can construct them on something curved like a saddle and so on—but that those geometries provide an equally good description of space, like Euclidian geometry.

So, in some sense the whole thing became a little bit like a game. I mean, I tell you the rules of chess, we play chess; I change the rules, we play a different game. So give me one set of axioms, it would give me Euclidian geometry; I change the axioms, it would give me another type of geometry, and so on. So in that sense this was really a revolution in mathematical thinking.

But the way that mathematicians in the end think about this is it is not that Euclidian geometry suddenly was wrong. Euclidian geometry remained absolutely true in the part of space, if you like, in which its axioms hold. It just got incorporated into a bigger body of geometries and so on, which, you know, eventually Einstein used and so on, and we use today, almost every day.

So what you said is true in the sense that, you know, we still calculate the area of a sphere, at least in Euclidian geometry, by the same formula that Archemides used in 200 B.C. We have not changed that. But branches get incorporated into bigger things and bigger things and bigger things, and mathematics becomes richer and more complex.

Levy: Does affect play a role in any of this?

Livio: What does—

Levy: Affect. For instance, you know, disharmony in music, when people first heard Schoenberg or the twelve-tone music of Bartók they were surprised. But now people would sometimes find harmonization to be a form of predictability, and they prefer cacophony, or they prefer this aleatory music. I mean the mentality changes—I mean if we're dealing with what makes us think about what's beautiful.

Greene: Right. No, I absolutely think so. That's part of I think the evolution that we're talking about. I think that's a perfect analogy.

But to go to the specific question that you asked, you know, in terms of personal moments, to maybe just spend one minute just giving you two. You know, in quantum mechanics, before I learned it formally I'd heard about this so-called many worlds interpretation of quantum mechanics, where there are many possibilities allowed by quantum mechanics, and the idea is that each one of the possibilities allowed would actually take place in a different universe. So if you're thinking about going left or right, in the actual biggest picture of things you do both. You go left in one universe, go right—when I first heard about this it felt like the worst possible theory I'd ever, ever heard about, so uneconomical, so laden with baggage, so much stuff being injected from the outside. But then when I formally learned quantum mechanics and learned the mathematics behind it, and learned that actually that interpretation of quantum mechanics is the tightest mathematical description, it's the one that has the least additional baggage, my view of it flipped completely. Because all of the sudden this was the one that didn't require extra assumptions, didn't require extra baggage. So it's still somewhat problematic, and it's still an active area of research, but that certainly was a moment where I flipped around.

The other direction was an interesting one where I was taking a course at Harvard, Math 106 with one Barry Mazur [laughter]—and it was a fantastic course, but I don't know if you remember, there was an interesting moment for me. It happened in your office—

Mazur: Oh, whoa! I don't remember this!

Greene: It was during office hours—

Mazur: Good.

Greene: We were doing Galois theory, and we were learning about field extensions and how to solve equations by adding various routes and so on. And you gave us a great problem, and, you know, I'd worked hard and come to the answer. It was, you know, various routes of unity and radicals and so on. And it just struck me, and I came to you and asked you, I said, "I've done all this calculation, but I feel like the way that we solve the problem is just by introducing a new symbol that solves the problem." And you thought for a minute and you said, "That's sort of right. That's really sometimes what it is." At that moment it just struck me that, well, at least certain mathematics, it just felt like it was just pushing around symbols, and they were important symbols and they meant something, but nevertheless it was just marks on a page.

And then, on the other hand, when you could use those marks on the page to describe the universe, that felt to me so much more compelling. And that was sort of a flip for me in a sense.

Mazur: Okay, well look, there's some high school math student put on the internet a joke, and the joke is so close to one of the deep ideas of Kronecker. Here's the joke, and then I'll tell you the deep ideas of Kronecker. The joke is, it's a piece of a problem, clearly a homework problem, it's say problem five. Then there's a right angle triangle, and the two non-hypotenuses are labeled, and there's an X on the hypotenuse. And the text by the X is "Find X." You can see the joke. There's sort of a scrawled ten with an arrow pointing to the X on the hypotenuse.

All right, now that's one way of viewing the symbol. Kronecker's view is slightly different, but very close. For example, if you want to solve the equation $X^2 - 2 = 0$, you might laboriously try to compute this, and you'll discover that it's the square root of two, and you'll discover that it looks like 1.4 if you're thinking of it—Kronecker says the essence of the solution of that equation is that whatever solution it is—whatever it is, it is a symbol, which has the property that its square is two. And if you're an algebraist, if you immediately use that—that is to say, if you so to speak retreat from the content a tiny bit, blind yourself to the content, and you algabrize the situation, you'll have a symbol, you don't know anything about it except that you know that its square is two. You will learn much more about that symbol, sometimes, than if you knew that it was 1.4, et cetera.

There's a literary analogy here, the Russian literary critic Viktor Shklovsky has a thing that he calls algabrization in literature, and what it is is having language retreat from content so as to produce the shape of things. At one point he calls it "putting the object in a velvet bag, so that you see its profile but you're not blinded to the particulars of the thing." I think that's what I was telling you in office hours.

Greene: Right. I should've stayed in mathematics.

Brann: Barry, as you know, there's a theory of the understanding of mathematics as symbolic abstraction in its very nature—that is to say, as a way of being allowed to say and to pretend to think things which you're actually not thinking but only saying. And that works. That's what's remarkable.

Mazur: That's algebrization, if you want.

Brann: Yes. Yes.

Nersessian: One of this—I mean this roundtable is called *Mathematics and Beauty*. It's not called something else and beauty. And over the years in conversations with mathematicians, and even in conversations with you, this idea of beauty as something that mathematicians are preoccupied with, connected to, feel empathy towards, or feel somehow they're work is more beautiful—I don't know. I'm still trying to understand why there is a particular relationship between mathematics and beauty. The examples that you gave, you talked about intuition, and you talked about the discovery of the proton after you've—well, we do that in psychoanalysis, but we never have roundtables called *Psychoanalysis and Beauty*.

Livio: Maybe you should.

Nersessian: Maybe we should. But, in other words, the notion of beauty in the way that mathematicians seem to see it doesn't enter into others, so what is that special connection?

Livio: Let me try my hand a little bit in this. In my opening remark I said that there are a number of aspects to this, so let me try first the simplest aspect. You have a problem to solve. Let's take a very, very simple problem, so that you don't need to be a mathematician at all to understand. I have a piece of chocolate. It has eighteen squares, let's say. How many snaps do I need to make to get the eighteen squares. You can think about this or try this and so on, but if you think about this a little bit you realize that every time you snap you get one more piece than you had before, which means that to get eighteen squares I need to snap seventeen times, because that's it. So this type of thing, when you realize this surprising way of thinking about something, this is one of the times that mathematicians would say that there is beauty in that.

There is a very famous theorem of Euclid that there is an infinite number of prime numbers—these are the numbers that are divisible only by themselves and by one—or, if you like, you mentioned square root of two, so let's—to show that square root of two is not a rational number—a rational number means there are two integer numbers, p and q, that square root of two is equal to p over q. There are many, many proofs of this, but I'll give you one proof, which I think I can say even in words. I don't need to write on paper. You say square root of two equals p over q, so p equals square root of two times q, and I square the two; I get $p^2 = 2q^2$.

Now, we know that every integer can be decomposed into a multiplication of primes. So p^2, which is p x p, each one of them is a multiplication of primes, so they all come in pairs, because there is p and there is another p, so they all have pairs; q^2 on the other side has the same, but there is the two which is unpaired, which means this cannot be. So it's a contradiction, and I proved the theorem. So, these types of intuitions, if you like, mathematicians see as very beautiful.

Now, one other thing I mentioned at the beginning was this business of passive effectiveness, and Brian talked about this a little bit. I was slightly surprised when Barry actually said that mathematicians want to explain things, because I thought that mathematicians most of the time don't want to explain anything.

Mazur: That's the job.

Livio: Physicists try to explain. I mean mathematicians—

Livio: So I want to give you a little story, okay? In the middle of the nineteenth century—

Mazur: As moderator, I'll separate these two.

Livio: In the middle of the nineteenth century we didn't have a clue what atoms were, and then came one physicist, Lord Kelvin, and Lord Kelvin was so fascinated by smoke rings—you know, that they are stable, they can vibrate—that he suggested that atoms are really knotted tubes of ether. Ether was that substance that was supposed to permeate everything. So in other words, every atom is some sort of a knot in an ether, so there is one knot that is the hydrogen atom, another knot is the oxygen atom and so on.

Now, in order to be able to classify atoms, to create something like a periodic table, somebody needed to be able to classify knots, namely to tell is this knot and this one, are they different or are they the same, and I can manipulate one into the other. And that was done, there was a mathematician, Peter Guthrie Tait, who sat down for twenty years and managed to tabulate all the knots with up to ten crossings in them—just up to ten, in twenty years it took. Can you believe it? I mean there can be knots with an infinite number of crossings. He only managed to classify all the ones with up to ten crossings. By the time he finished classifying up to ten crossings nobody believed anymore that's a model for the atom.

So you might've thought, according to Barry, that at that point mathematicians will leave the subject, because the motivation was no longer there. No. This is exactly when they got really interested in knots.

Scarry: But you know—

Livio: And they started working on knots and so on, and then for example in 1984 a New Zealand mathematician, Vaughan Jones, who works at Berkeley actually, discovered something which is called the Jones Polynomial, which is, well, like an algebraic expression that's a bit like the fingerprint of a knot. It identifies a knot in a very nice way. But all of this was done with no application whatsoever in mind.

And then a number of things happened. One thing that happened, biologists by then already knew that the DNA is a double helix and so on, and when cells need to divide then this double helix needs to unknot itself in order to copy itself, and then it knots itself back and so on. And so that they needed—and enzymes do that work. Enzymes pass one strand through the other and so on and so forth. They discovered that there are mathematicians that they would show them two DNA configurations and the mathematicians will tell them at what rate the enzymes do the work, because they knew all these operations of transforming one knot to the other.

More than that, this Jones Polynomial became suddenly very important in string theory. The interactions of the strings can be explained, but—so this is a sense of *incredible* beauty. I mean here is something, you know, people were doing it for fun, if you like, and suddenly it turns out to be—

Levy: That's the definition of Philoctetes, by the way.

Livio: —explaining something so fundamental as interactions of strings. There is really, there is nothing more beautiful than this.

Mazur: There is something very interesting—I mean, you're right, it is fun. Knot theory is fun, and it's magnificent mathematics. But it is the basis of three-dimensional geometry. I mean its the essence of three-dimensional topology, and anyone who has to deal with three-dimensional spaces, which are *immensely* complex, will be looking for that concept, which in number theory is very much like prime numbers. And the concept in geometry that corresponds to prime numbers is knot. So just as one is fascinated if one is doing numbers, fascinated by the series 2, 3, 5, 7, 11 and 13 and so on, as the building blocks of numbers, so you would have to be dealing with—in other words, it is fundamental.

Livio: I agree.

Mazur: And the fact that it applies to string theory, well—

Livio: Is an added bonus. Is an added bonus.

Mazur: What?

Livio: It's an added bonus.

Mazur: No, I would expect it to apply to anything that gets to geometry in a fundamental way.

Scarry: But some of the description you were giving, you were answering the question why math has beauty in a way that you're saying psychoanalysis isn't so usually connected with beauty—

Nersessian: Well, we've never talked about psychoanalysis and beauty.

Scarry: Right.

Nersessian: But mathematicians have something with beauty for some reason.

Scarry: But you know, this example of the square root of two, I remember once reading an essay—and now, this is twenty years ago I read the essay, so anybody who's read it more recently can correct me, but it was by the mathematician Seymour Papert, and it was trying to describe the aesthetic pleasure of math, and I think he was using solving for the square root of two, or maybe it was the square root of minus two, but he described the way in which the two keeps appearing and disappearing on the other side of the equal signs, and sometimes as a superscript and sometimes as the number by which you have to multiply things. And he was arguing that it had to do a lot with a sense of play, and almost a kind of theatrical play, where things are making entries and exits that you didn't explain, that you didn't anticipate, which does actually coincide a little bit with your description, and with your description of the surprise at making the snaps in the chocolate.

But, here's my question: why wouldn't something like—and I don't know anything about psychoanalysis—but why wouldn't that description also be, you know, why wouldn't it travel out to other realms as well?

Greene: I have a thought on that, which I think perhaps it really does and it may just be cultural-linguistic difference in how the terms are used. But I think at the base of all the examples that are being given to me is the following experience, and the experience is you go from utter confusion suddenly to complete clarity. And it's that move, where you're in this fog of not really understanding how many snaps of the chocolate, or understanding Galois theory, whatever it is, and all of the sudden the pattern becomes clear. I mean ultimately we're pattern seekers, and you're in this midst of not understanding, and all of the sudden by virtue of thinking about things a little bit differently it snaps into clarity. And I think that is sort of the moment where we say, "Wow, that's beautiful." And I think that probably happens in a lot of other fields too.

The one thing that for me is different, and I think this is really ultimately a personal choice, a personal aesthetic sense, I'd like to have these moments, and the ones that are most meaningful to me are when I feel like the pattern that's emerged is—you know, this perhaps contradicts what I was saying before, but has a sense that it's eternal. Now, I know that it's not, but it has a sense of going beyond earth, going beyond human society and touching something that transcends it all. And, yes, there are aesthetic senses that will evolve over time, and what is a good theory today in physics probably isn't going to be a good one 10,000 years from now. But you have the sense of touching something beyond the every day, and it's when you go from that confusion to clarity in a domain where you feel like you're touching something beyond the ordinary that that is a strong sense of beauty.

Brann: Can you explain—it seems to me this has to be somehow involved in trying to get at what beauty means in subjects which are essentially thoughtful. Why is it that in explaining what it is that gets to you, you're compelled to use sensory metaphors all the time? Why is that? Clarity itself is—

Greene: Well, I mean the subject to me is very sensory oriented—

Brann: And yet it isn't something in the senses—

Greene: Well I think it all depends on your style. I mean there are some people, and let me just speak in physics, who really are very equation oriented. Their understanding and their process is totally about doing the calculations. And there are other for whom that is more a tool, and they aligned with it have a parallel picture that's running that gives them an image of what's going on. And I veer more towards that. There are others who are far stronger than I am technically and perhaps can get to an answer that I can't get to, but for me the sense of understanding isn't enough if I can do the equation.

Brann: In physics the answer may be more compelling, but what about mathematics? Mathematicians talk the same way, and yet they're not really talking about—I mean clarity has to do with light, right? And touching has to do with tangibility. Why—and I'm merely asking the question. I'm wondering what there is about thinking that is almost unavoidably sensory? There doesn't seem to be such a thing as merely thinking, right? Unless it's cranking. Why is that? I mean what is it in us that connects our sensibility and our thinking?

Mazur: I think it's—it gets to the question of why we think of mathematics, which has really abstract concepts, if you wish, at the base, as something that allows us to see things. I mean it gets back to your mentioning that visibility, taken in some metaphorical sense—

Brann: But it is a metaphor—

Mazur: It's a metaphor, but we have no other vocabulary to say these—to describe these moments that Brian just described, of going from confusion to—

Brann: To clarity—

Mazur: Well, to light.

Brann: Yes, to light.

Mazur: And you might say why don't we have other words for it, and it might be for some deep reason that there are no other words for it. I mean that is—it may be stronger than the usual metaphorical use of, "I see."

Scarry: I was going to say, I agree with Barry that you've circled us back to your initial comment about visibility in *The Phaedrus*, and I don't think that Plato means that to be metaphorical, because he says the special generosity of beauty is that it does make itself available in the sensory world. And I know that a lot of mathematics and physics is absolutely beyond the sensory horizon, but these moments that are being described are always moments when it suddenly has come within the bowl of perceivable space as though it were this round table.

The only other thing I can think of is that—no, go ahead—

Brann: What you're saying makes so much sense, and yet when you drive it to its conclusion it means that when mathematicians speak of beauty they're speaking of *falsity*. Because that's exactly not what they're really doing.

Livio: You know, Einstein once was asked in an interview—you know, Einstein was notorious for making many mistakes. He kind of knew what the final answer should be, but along the way he would make lots of mistakes, actually. His papers are peppered with mistakes. And he was once asked later in an interview how is it possible that he still manages to get to the right answer at the end, and he said—I'm now quoting him—that he was always trying not to think in words at all, because words lead you to possible contradictions and things and so on, and he just thinks in images, and this leads him to the right answer. So that, in a way, is what you have been saying, I mean, you know, that it's these images that drive us.

So even mathematicians, even when they work with the equations and so on, they probably, there is some sort of image that drives them. It's not maybe the thing that is exactly written on the paper at that very moment.

Levy: It's a priori though, isn't it? Aren't we talking about mathematics predicated in synthetic a priori kind of knowledge that is in the brain and laws for which you don't need an empirical—

Mazur: Well, one of the—let me first respond to—one of the big changes in mathematics occurs when it's a viewpoint that's changed. It's a whole way of—I hate to use the word seeing things, because it's begging the question, but that's the only word that comes to mind. But I'll use the word viewpoint without emphasizing the 'view' aspect of it. It's finding a perch so that when you see—what you see is a much larger terrain. And you see it clearly. I mean: to explain means to turn to a plane in some sense, and you see things in a way—okay, what is vision? Vision—

Nersessian: But that must be what you mean by beauty, because what you described, the first part, everybody has that. You're confused and you discover something and suddenly a lot of things make sense. We have that regularly. But there's something more you said, and that must be then why mathematicians talk about beauty, what you just said.

Mazur: It could be, that you—that there's a panorama that you didn't imagine that you could see, and now you see. It's like coming to Kansas after the east coast or something.

Brann: Let me say why it seems to me that this—

Livio: That's beautiful?

Mazur: I was kidding, I was kidding. It is beautiful actually.

Brann: —a kind of self-delusion, even a falsification. People will say when they're studying Lobachevsky, "Yes, I can see it." And they draw it. They go to the board—you know, our students will study Lobachevsky, and they go to the board, and now they draw two parallel lines that meet. And really what are they drawing? One straight line and one bent line. That's a falsification. They mean a straight line, but in order to see it they have to draw a false one. Or they say, "Oh yes, you can do it on a saddle or"—

Greene: That's the limitation of the blackboard.

Livio: Yes. Yes.

Brann: You can do it on a saddle.

Livio: Exactly, turn the blackboard into a saddle, they would have no problem.

Brann: What are they looking at? They're looking at a piece of Euclidian space. They're not seeing—

Livio: They're trying to project.

Greene: Yes, I agree.

Brann: You agree. So—I mean so what is it that it takes to say that mathematics is beautiful? It takes a kind of devotion to the same metaphor that philosophers employ when they say, “I see,” when they mean—what do they mean?

Mazur: I think we’re focusing on this—

Brann: Yes.

Nersessian: They think they see, but the mathematicians actually see.

Livio: I think by the way that part, or maybe part of the reason that you in psychoanalysis—I don’t know, at least the things that, when mathematicians do something and when theoretical physicists do something, I think at the end of the day the places when we say that things are beautiful are those when what we deal with deals with something that is truly fundamental. It’s the words that Brian uses, that it has—you know, it’s forever. Even if it’s not forever, but if it deals with something that is truly fundamental, you know, this is what drives the universe at large and the universe at small and so on, these types of things.

When we deal with much more complex phenomena, when fluid dynamicists solve the equations of fluid dynamics, I don’t think you hear the word beautiful so often. Maybe you hear it when they discover something like a Feigenbaum number, which is some sort of universal numbers that appear in all transitions to turbulence or something. But when they just solve something that’s really complex, beauty is not mentioned as often as that. And psychoanalysis, by the nature of the beast, I mean it still deals with very complex systems and so on, so maybe this is why—I’m just venturing here a possibility.

Mazur: Well, I don’t know, there are some famous problems about turbulence, or famous problems about solutions of physical equations—

Livio: They are the ones that deal probably with the more fundamental things, as opposed to, okay, I just also solved this stream hitting this wall, and I don’t know what.

Mazur: I think you’re using the word fundamental in such a general sense that it really does correspond to what mathematicians value as well. That is, it doesn’t have to be a fundamental fact of physics to be fundamental. It could be a fundamental way of thinking.

Livio: Yes—

Mazur: Could it not be?

Livio: Yes.

Mazur: And so unfortunately we now have two words. We have ‘visible’ and ‘fundamental,’ and there’s beauty sort of sitting in the wings there, and to some extent beauty is attached to things that are fundamental. Beauty is attached to things that let us see far and wide at the same time. And I guess one of our problems in discussing it is we don’t know to what extent the metaphor can be sharpened. Isn’t that the issue? That’s your question I think.

Brann: That's an issue, yes. Yes.

Scarry: Well, one way in which beauty is fundamental in non-mathematical areas—I'm also aware that we said we would open things up to the audience, so—

Mazur: I think—yes, maybe we should—

Scarry: Maybe we should just—

Nersessian: But finish with your—

Scarry: Okay. Well, I was just going to say that in—and this might not apply to math, but in literature and philosophy often there's a claim that there's a kind of life pact between the beautiful thing and the observer. And I mean Diotima told Socrates who told Plato who tells us that when you see someone or something beautiful, or if you see somebody beautiful it can give rise to the desire to have children or to write poems or to write legal treatises or philosophic treatises, and there are many later statements that talk about this kind of generative impulse. For example, Augustine talked about music as a life-saving plank in the middle of the sea, and Dante wrote a book called *The New Life*, and so on. And, you know, there's this—even though there's lots of variations in poetry over time or artworks over time in what is being validated, there often is this kind of affirmation of this basic fact of aliveness, and so I was wondering if something like that could be at the heart of mathematics as well.

Mazur: You said being alive, in the mind—

Scarry: Yes, or just—I mean just the—of course if you see something beautiful, let's just say the autumn leaves today or something—

Brann: So you're saying that beauty can be arousal.

Scarry: Yes, that it is pushing perception to the higher level of acuity, and conversely it instigates in you the desire to carry that thing forward in time, to protect it and nourish it, or make sure the theory, the account gets out to other people. So in that way it could be—

Brann: Yes, I mean if I wasn't ashamed to do it I'd ask the people, the mathematicians here whether there is an erotic element to doing mathematics.

Greene: I'm glad you're ashamed to.

Mazur: I think on that note we can open it to questions.

Levy: If you want to ask a question you have to come up to the mike. We have to capture you.

Audience: But it seems to me the last minute or two of discussion seemed to really get to, at least from my perspective—

Levy: Could you say who you are? Everyone say who you are.

Audience: Michael Sachs—really get to the heart of the matter, because the one thing that you really haven't been touching on, although you've been going around it, is the emotions that accompany the thought that something is beautiful. Now, obviously in each one of your fields you're very passionate people about it, that when you do see something as beautiful, there's an emotion that accompanies that, and that goes with the solution of a problem, bringing things into visibility, and I even think in a psychoanalytic realm, if you were to talk to a psychoanalyst who's dealing with a patient and all of the sudden there's an insight there that came, that would be accompanied by an emotion, and that emotion would be a very pleasurable emotion, which gets back to Eva talking about why do you always talk about it from a matter of sense, why are the metaphors so sensual, because that's the feeling that accompanies it. And I think what you just said about the idea of being alive, this is a—that feeling is really fundamental to existence. It's certainly a craving, that feeling, so that the kind of—when you talk about labeling something as beautiful, it's almost like falling in love. You really can't put it into words, but it's there, and you know it, and it's positive.

Livio: I actually agree very much with what you say. And what you said also reminded me that, you know, we are now fortunate that people can do things that in previous centuries they couldn't, namely they can do functional MRIs of brains of people who do various things. And they did find that when people see things which they describe as beautiful, the same area of the brain is activated as the one that is activated by addictions, for example. You know, if you're very hungry and you suddenly get food, or if you need drugs and so on, it's the same area of the brain.

So I think you're right. You're probably right, and we probably also have the same thing as we reach some certain solution and so on in this. And of course the oxytocin is then flowing and things like that, and so on, yes.

Levy: Tell us your name—you know, I like to get to know people—and tell us what you do, or what your profession is.

Audience: Retired teacher—

Levy: Okay, go ahead.

Audience: I noticed one term that hasn't been used that mathematicians use quite often; it's the term 'elegance.' Now, it hasn't been mentioned at all, and that surprised me. And what I want to know is can a mathematical or physical theory be elegant and beautiful if it's counterintuitive to the way our mind works, in particular quantum theory?

Greene: Well, that is a word I'm fond of. And I do think in fact some of the most powerful moments where that word would be relevant would be exactly the ones that you're describing, where, you know, you were talking about that switch from seeing things one way to seeing things another way. The most spectacular moments as a physicist, and I'm sure in any discipline it's the same, are the ones when you learn something about the universe that's verifiable, experimentally or observationally verifiable, that's completely counter to the way you would have thought things work. And quantum mechanics is the primary example of that, so to me quantum mechanics is the most spectacularly beautiful, the most fantastically elegant structure that our species has ever created, because it is so completely counterintuitive it is absolutely nuts that it's right. And that is spectacular.

Audience: But what about the conflation between general relativity and quantum theory?

Greene: Well, that is a thorn in the side, isn't it? And that's what gets you up in the morning and keeps you going. And so that's a great thing to have. I mean if everything was solved—you know, it could've been the case that the world, say was just Newton or just quantum mechanics. It could've been that way. And I have to tell you, I thank—I don't know, God, or whatever—you know, I thank the fact that the world is not just Newton or Einstein or quantum mechanics because it gives us something to do. And it's such a fun thing to do. But it could've been simpler, and it could've all been solved. And thankfully it hasn't.

Audience: Thanks very much.

Audience: I'd like to ask a question linking to the visibility point. You mentioned knot theory, Galois—I'm very curious about the role of nomenclature in the way of shaping thinking, because the symbols you use to express things might influence the way you choose to [inaudible]—

Levy: Can you speak into the microphone?

Audience: —your book on symmetry deals with that question explicitly. So I'd just like to get a sense of what role the nomenclatures you use and choose play in your aesthetic sense, but also in solving the problem sense, if there's any kind of an alignment there.

Mazur: Can I answer that? I mean two of the most brilliant pieces of nomenclature that mathematics has, both of them are due to Leibniz. One of them captures in a symbol, which is almost a Chinese hieroglyph, all of differential calculus, and the other captures, in another Chinese hieroglyph, all of integral calculus, or a good deal of it, where the rules for manipulation of the symbol are somehow packaged in the symbol. So the standard—well, the standard way of talking about a derivative—a derivative is you imagine that you're trying to compute the instantaneous speed, say of your car, of your going, and the way you do it is you would approximate it by computing for a certain time interval how far you've traveled, and then you take the ratio of the two. Now, that ratio is not the instantaneous speed at any point that you can name, but rather is an approximation, so to get something more and more exact you have to take finer and finer time intervals, smaller and smaller time intervals and take a sequence of ratios which you hope will converge to an actual number, which we'll call the instantaneous speed.

Now, that number is not a ratio of anything, because you're taking a ratio of fractions where the numerator and denominator are going to zero. So if you thought of that number as a ratio it would be zero over zero, which is nothing. Nevertheless, Leibniz used fractional notation to describe this. He called this DF over DT; DT is change of time and DF is change of distance, let's say. And it's not a ratio, it's not a fraction; it's something that is described by sort of a theory, if you wish, and yet contained within it is the seed of the entire theory. It's a mnemonic of how you got it and it's a mnemonic of how you're going to use it, because it sometimes does play the role of a fraction—and the same with integral—

So, in some sense the—

Audience: Was that beautiful when you got it?

Mazur: Well, I think it's not only beautiful topographically, but I actually think that it's conceptually sort of a beautiful move, and also a prod. It says, "By golly, when you define something your terminology, your notation is going to have to do work in the future, and you should fashion it to be as compact as possible, as mnemonic as possible—it tells you exactly how you got it and how you're going to use it." And, yes—no, it's—

Brann: I can think of a case in chemistry, the beginning of chemistry I think was in the renaming of the cabalistic terms for substances, for terms that suggested atomic structure, like hydrogen, which means essentially something that gives birth to water. So that's another case where what they themselves called neologism made all the difference.

Nersessian: Before you ask your question, I wonder if Sarah Ferguson wants to say something about beauty and math?

Ferguson: Well, yes. For me—

Nersessian: Could you—

Levy: Come up to the mike—

Nersessian: And then you can ask your question. It's just because she has done work here and she has—

Ferguson: For me I think, as a former mathematician, language was really the medium for me. I'm now a painter, so I think of a theorem—I can draw a parallel between a theorem and a painting—which means not all theorems are beautiful.

And then the proof, the language of the proof is the process, or what reveals it. And there is a falsity, I think, in terms of sometimes an elegant proof is one which conceals. It just does it in an economy of means. In mathematics it was the language always that carried the aesthetic value. And so I did—my first body of work was drawings of what I call the math heads, where I covered them in computations from my research, because that's really what I did. Instead of painting I was at a desk with tons of paper and pens, and I just wrote and wrote and wrote, and so for me it was a very physical thing.

Mazur: Thank you.

Audience: Okay—

Levy: Tell us who you are and—

Audience: I'm Susan Donovan, and I'm a graduate student at Hunter. What I wanted to ask was is beauty for physics and mathematics necessarily always helpful? Because I think about how Galileo thought that if you had a chain and you were just letting it hang like this, at first he thought that was a parabola, and it's not a parabola. It's another curve that's based on trigonometry. And it would be kind of more beautiful maybe if it was, because parabolas are so simple. That's that reductivism you talked about. So what about that other side of beauty, when it leads you in the wrong direction, because you just want it to be simple?

Livio: So, that can happen. It can happen. Sometimes later we realize that what we thought was not beautiful actually was a misunderstanding. [laughter]

Greene: That's the evolving sense—

Livio: Yes. So I will give you an example based on the same thing that you just—not on the chain. The chain was solved, yes, by Bernoulli and others, but also from Galileo. Galileo absolutely dismissed the idea—he was a correspondent of Kepler, and Kepler discovered that the orbits of the planets are not circles, that they are ellipses. And Galileo couldn't believe that, and the reason he couldn't believe this was exactly the reason that you said, because circles were supposed to be these absolutely perfect shapes, and therefore how could the orbits of the planets be anything other than those perfect shapes?

Now, here is where later we understand that there is a misunderstanding. Galileo also had in mind the symmetry, but he thought in terms of a symmetry of a shape. A circle has this symmetry that you rotate it however you rotate it and it looks the same. Instead, what we discovered later was that Newton's laws, and the laws of physics in general, have this symmetry in them on the rotations, but it is a symmetry of the law, not of the shape—namely, in the case of the orbit, the orbit can be an ellipse, the shape is not so important. But that ellipse can have any orientation it wants in space, and all of those are allowed orbits.

Now, had Galileo realized that that's what he should have been thinking about, I believe he would have agreed that this is even more beautiful than the circle. But at the time he thought about the shape, the symmetry of the shape.

So, occasionally we think that, oh, it should be like this. It turns out to be different, which looks less beautiful, but then our sense of beauty also evolves. We understand that there is a deeper sense in which we still regard this to be beautiful.

Mazur: Okay, so next question?

Audience: I used to be a student of Miss Brann's long ago. And I have a question about her citation of *The Phaedrus*, because I came half expecting today to hear a lot about music, and instead heard nothing. So in citing Plato about beauty being visual, I wonder, well, what happens to music in that definition? Is it left out, or is there some way of reading his words to accommodate for that?

Brann: Well, I'd have to say that the first great mathematical theory, which is that of ratios and proportions, was in fact a musical theory. That is, the Pythagoreans discovered that consonants had to each other small number ratios, and that the composing of those ratios resulted in the diatonic scale, so the very first great mathematical theory has its very definite beauty of an audible one. So I'm glad you brought it up. I think you're showing your education here.

Scarry: And I think in a way that Plato—I mean I don't know what you would think Eva, but I think that Plato's actual term at that moment is, "Beauty is clearly discernible."

Brann: Yes, something like that.

Scarry: And I think he means it's within—it becomes within the sensory horizon, and I don't think that that would mean that it has to be visual as opposed to auditory—

Brann: No, he doesn't mean visual that way. No, he means—

Scarry: Yes.

Audience: I'm Lynn Gamwell, I'm a historian and a writer. And many of us were here a couple of weeks ago, we did a program on Mathematics and the Divine, and heard about the traditional association of mathematical objects with religious issues, because they exist outside time and space, and they're perfect and eternal and so on. And I wondered if our mathematicians and physicists could comment on the issue of the difference between when you come to a realization between revelation and reason.

Mazur: Well—Brian? Brian already invoked a vocabulary that is almost—

Audience: Revelatory—

Mazur: —of that spirit—

Audience: That's what prompted me—

Mazur: That is to say that one is looking for universals, things that are so unbound in culture, so unbound in—independent of language, independent of culture, independent of time. It's the sort of ideas that anyone who meanders through the cosmos will eventually come across.

Now, I don't know whether you need to put a theistic overlay on that, but it certainly is kind of a companion thought to some of the more religiously described, sort of revelatory notions about the natural philosophy. I don't know whether anybody else wanted—

Greene: But I think the power of the experience, at least the one that I was alluding to, is it's not just the moment of revelation per se, but it's the fact that that moment of clarity is distinctly founded upon a chain of reasoning and explanation that gives you a complete sense of understanding, as opposed to acceptance. And that's the key thing, that you can see the full chain, from something that is so basic that you feel it almost doesn't need explaining, through the chain of explanation to this completely unexpected place. It's not just the unexpected place coming down on high and imposing itself upon you as some revealed truth. It's the fact that you understand the linkage by which you believe it.

Audience: Right, you've prepared yourself.

Greene: And itself comes in a package, where you only have a sense of the power of the moment because you can see why it's true, not because someone tells you it's true.

Audience: Exactly. Yes.

Livio: And, I might add that you also know that whatever idea or theory or whatever you came to, you only consider it to be an acceptable theory if it is actually also falsifiable, that you can actually make predictions based on that and you can test those predictions in the future.

Audience: Can I have one comment too? Unrelated—just to remind us, with the issue of visibility, of Plato's metaphor of the cave, of the sun makes the visible world visible and the way in which the form of the forms makes the mental world understandable—

Brann: Yes, it's in—

Audience: The Platonic context, yes, right.

Audience: Okay, I'm a conceptual artist, my name is Olga Ast. And in art we already really don't like things that are too beautiful. Too beautiful for us is a negative term. And what we usually say what we like is great work, is great idea, is great solution—great solution we don't say, but actually it's very close to your definition of great solution, of new opening, what we see in this. When it's too beautiful it's already on commercial side, it's—you could suggest it to the general public and say, "Oh okay, you can buy it. It's really beautiful."

But my question is actually to psychoanalysis here, because once I talked to several people about also beauty and science, and a lot of people are saying that beauty is something that you enjoy. But it's not really correct, because you could for example be scared of a hurricane but consider it very beautiful. So in this discussion, kind of after this discussion I came to conclusion that beauty, you probably should treat a sense of beauty as a separate emotion. And if it's a separate emotion, then it's really subjective, like that perfect symmetry is beautiful, but now they don't consider perfect symmetry as beautiful. It's actually almost symmetry is probably much more interesting and beautiful for us.

So my question is should we consider the sense of beauty as a separate emotion, like for example suffering?

Nersessian: Well, the problem is that when we mean, in the sense that you describe beauty, it also could be said it gives you some sense of pleasure, and so it falls under the general category of pleasure, just as suffering is on the other side.

My impression was, and the question I was asking earlier on, which I think I have some answer to, is that mathematicians mean by beauty something beyond just this subjective sense of something is beautiful, and that that aspect of it that is beyond—because as I said before, there are times in my work when something comes out of me as an intuition. I have no idea where it comes from. It ends up being so correct that it opens up a number of things that up to that point I had not seen, and my patient had not seen. But yet I don't end up that session thinking, "Gee, wasn't this beautiful." So I have a feeling there is something more to what they mean by beauty than this kind of subjective feeling of enlightenment, insight, good—just a sense of well being. They mean something more.

Scarry: Could I protest briefly, the account of art you gave, which is a view you hold and is a very reputable view held by many major artists, but I want to contest it. And for many decades beauty was banished from humanities departments in universities, and also from museums and also from art studios and also from architecture schools. And in all these sites lots of people now say that's wrong, that beauty needs to be credited, because what we've all been looking at from the past are very objects. And I mean it's not that one of us is right and one of us is wrong, but I just wanted to put the alternative view on the table.

You know, your worry about the commercial I think follows from the fact that if everybody vacates the field and won't use the vocabulary beautiful you can be sure advertisers will still use it, and that leads to the mistake of thinking that if you're suddenly talking about something beautiful you're talking about something commercial, or, to put it another way, when you see something beautiful the only response is to buy it, rather than, as we've been talking today, if you see something beautiful it's the time to intensify your research, or it's a time to educate yourself still further. So it's not something we could solve right now, but I just wanted to state the alternative view.

Audience: Hi, my name is Liam Cohen, I'm a freshman in high school. I also have a friend who can't really be here right now, so I'm going to ask some questions from him too, just one.

So, basically I've been listening to this roundtable, very interesting, and I'm very happy to be able to ask this question from such great minds. But what it is is, what I understood from this is that beauty is really in the eye of the beholder, so each different mathematician and physicist here has a different view of beauty and how mathematics and physics and bioengineering, how that all relates. And my personal opinion, I feel that beauty and mathematics, in relation to mathematics is that mathematics is this bridge between actuality and imagination, and when a physicist theorizes something, or he thinks of something that—

Mazur: Or she—

Audience: Yes, or she. Of course. Thank you. When they theorize something they have to use mathematics to prove that. So I see mathematics as this sort of token or bridge, and this way to explain the universe, and I think of that as beauty in a sense. Whereas when I can truly relate imagination and actuality, that to me is beauty. That's just my personal opinion. And I was just wondering how you guys all felt about that, and just how you think of mathematics as a bridge between imagination and actuality.

And also, my friend's question is—he's saying as we all know, the solutions to everything in life are infinite for the art of mathematics. But how do you think these advances can be made nowadays or in the near future?

So those are my two questions. .

Brann: If I may. You said the question what is beauty seems to have a number of answers. It seems to me there's a distinction that might be interesting to you. It's one question to ask what things are beautiful, and I'm always amazed at how much unanimity there is about that, what mathematical theories, what physical theories are beautiful.

The other question is what does it mean to be beautiful, and that's where people differ. That's what makes the question interesting. If they were utterly—had utterly different answers to the question what is beautiful there wouldn't be much to talk about. But there is something that they regard as beautiful, and that's what makes the question interesting—namely, what is it that makes it beautiful?

I want to say one more thing if you'll let me—

Audience: Of course.

Brann: You said that's just your personal belief. Never say *just* your personal belief. That's all you can have is a personal belief, and it's not a *just*.

Mazur: Mario?

Livio: Concerning the bridge thing, it's actually a very deep question. I don't think I will have time here to discuss it, but Galileo for example is the person who first wrote down a statement which said that mathematics was the language of the universe. In other words, the universe is written in this language of mathematics. When you think about this from today's perspective, that's a little bit ambiguous, this definition, because for example if mathematics is really only an invention of the human mind, how did the universe know to be written in this language which is an invention of the human mind? So it becomes quite complicated.

And the answer is that mathematics is a bridge. I think the word that you use is a very good one. It's a bridge. But the bridge works in somewhat complicated ways, because on one hand we invent mathematical concepts, then we discover relations among them, and then we apply those things as models to physical reality. And when that works we're very happy. When it doesn't work we actually try something else. So there is a certain natural selection and evolution, if you like, there in the solutions that we apply, and we constantly improve the language that we use. So it's a bridge, but it's not a fixed bridge. I mean it's a bridge that we work on all the time.

Mazur: It may be a bridge we travel on as well. I mean that imagination can get closer to the actual is amazing, after all. And your image there is quite apt, that imagination needs something to apply itself directly to the largest realm it can possibly apply itself, and for that it needs conveyances, like bridges. And perhaps mathematics is that from that point of view; it's a way of conveying the imagination to its broadest reach.

Audience: Thank you.

Audience: I'm nobody, and I'm doing nothing.

Levy: But you do have a name.

Audience: No name. I'm trying to give that up too. My question is kind of a spiritual one or transcendental, which I think you touched on and you touched on as meaning. And what comes to mind listening to you all is St. Thomas Aquinas, who had three functions of beauty, and I wondered if you would relate to that. He said it had to have wholeness, a harmony and a radiance. What do you think about that?

Scarry: I mean—he used that language, he also used integrity, proportion and claritas. And some of those, I think that claritas, or the radiance, is one that we have been talking about all along.

That is, the ability of something to just stand forth and be available, whether it's in the "aha" moment or whether it's in a more serene moment—

Audience: Of course it could be energy for the physicists.

Scarry: Yes. And I think that proportion, or harmony, is audible in the times when our discussion has touched on the idea of symmetry or equality or ratio. And wholeness may be when we've touched on the idea of the aspiration for things that are fundamental—but I'm not sure. Maybe that's the one that we've talked about least.

Livio: But also to explain with—as little as possible to explain as much as possible.

Scarry: Yes. Yes, that's right.

Audience: Do you think that could be transcendental, or transformational for us and for you? I mean isn't that what you probe these questions and live this life?

Scarry: Well, Aquinas was, at the moment he gave that three-part definition he was describing the face of Jesus, if I remember correctly. And he was making a larger account of what things are done by the Father, what by the Son and what by the Holy Ghost. And, you know, maybe this goes back to the earlier question about mathematics and revelation, and the intuition that people have that there is something—you know, quite apart from whether one has any specific religious belief, whether one has some sense of a magical underpinning, or a kind of unifying presence.

Audience: Thank you.

Audience: I'm a neuropsychologist, and I can't help but offer a little bit of the thinking that someone from my field would go through in listening to this discussion. And in answering the question about beauty, some of the neuroscientists have looked at what people actually found as beautiful, and they found that we have like a running average, a sense of a running average of what the norm is, and beauty is a deviation of that average in a certain direction. So when we think about facial features, it's larger eyes or smaller nose or bigger lips or whatever, and Barbie doll as an example of an exaggeration. A caricature is often considered more like the thing than the real thing. So it was interesting to me that Brian's point about asymmetry is beautiful, because the asymmetry in the universe accounts for the universe, and asymmetry is what we pick up on, but we pick up on it in a given direction.

The other point I wanted to mention is about the difference between the visualization and the bridge. We understand now that the right side of the brain is nonverbal, intuitive and forms hypotheses about the world—you know, what Brian was saying also about visualizing the solution without having the words for it—whereas the mathematical side of the world, which is the left side of the brain, comes up with the language. And the two are both working in their separate processes, and then at some point there's an "aha" moment, where now I have the words to say what I've been thinking. It's the proof. And we might experience that as a form of beauty, but it's really an alliance of two forms of knowledge that are different from each other but suddenly come into existence at a certain moment.

Mazur: Can I say that I agree with you entirely, except that mathematics would co-opt both sides.

But there's this, let's say, companionship between geometry and algebra, which began with the Pythagoreans and is continuing today. It's deeper and deeper and deeper. There are two intuitions. One is the geometric, visual intuition, and the other is, as you say, you described it beautifully, a verbal, algebraic definition intuition. That there are profound connections between these two, this is an unfinished business even now. I mean we will for the next fifty years be seeing more and more fundamental relationships between these two ways of thinking, and each one nourishes the other.

Audience: Right, right.

Audience: I am a science reporter, and to some extent I specialize in disputes among scientists as to general beliefs and results. And I've been looking at the literature of CERN's Collider, which is about to start up, supposedly in a month or two, in Switzerland. And as probably many people here know, there's quite a bit of discussion about whether this will end up creating black holes and strangelets which may or may not reduce the planet, either to a molten asteroid the size of a football field, or else maybe even reduce it to the size of a marble and it will disappear down one of these black holes. And I've been looking at the literature of this, and I've noticed that it seems to be in effect undecided as to what really will happen once they crank this accelerator up, up to levels that—you know, I think it's four times as high as previously. And Brian Greene wrote this rather gung ho piece in *The Times* a year ago [laughter], saying we should go full speed ahead with this thing, and he offered an argument which has since been admitted by CERN as not any good in terms of defending the safety of this operation—

Greene: Which argument was that?

Audience: What was that?

Greene: Which argument are you referring to?

Audience: The cosmic ray argument, saying—

Greene: The earth atmospheric cosmic ray argument, or the more general one?

Audience: Well, let's put it this way: the argument that because cosmic rays are constantly hitting the earth at high speeds and—

Greene: That's only half the argument. The other half is equally important. You can't take half away and—

Audience: Good. I'd be delighted if you'd just expand on that. But I just wanted to say to you, what I wanted to ask you, in the current context, you've hinted at the fact that if something is beautiful, therefore it's probably more likely to be true than not. And you've also said that we have theories of physics today which, you know, will be probably replaced in 10,000 years, and therefore viewed as wrong, or inaccurate, and I wanted to know, since it seems somewhat undecided as to what will really happen when they turn on this accelerator, and there are very famous, or at least very reputable scientists who argue very well that it is very dangerous—and apparently their papers are not being read by physicists involved. At least two of them I've asked at NYU had not read Plager's third edition of his paper, which he brought out a month or two ago. I wondered if we could say, well, we don't know what's going to happen, because we've never been into this uncharted territory, but the critics theories are less beautiful than the theories of the people who support going ahead, and therefore we can be confident on that basis.

Greene: That was a long preamble, but, yes, sure. You know, I don't think that you ultimately want to judge—are you going to take my picture while I'm answering? Okay.

You know, I don't think you want to judge the correctness of theories by any of the aesthetic considerations that we're talking about here today, because ultimately what will determine whether a theory is right or wrong are its predictions and whether those predictions agree with data. And I don't care how beautiful a theory is, if its predictions don't agree with data it's wrong. I don't care how ugly a theory is, if its predictions agree with the data then I'm willing to accept it as correct.

The arguments that have been made for the issue that you're describing are not based on beauty. They're based upon calculations, using our best understanding, as you say, which could change, of how the world works. Now, I should say, based on our best understanding of how the world works there is a small chance right now that a dragon will appear in this room and swallow us all up, and the earth as well. There's a non-zero chance according to quantum mechanics that that will happen. I don't worry about it. And I don't advocate that anybody else worries about it either, because the probability is so fantastically small that it isn't worth worrying about. If you worry about that you should worry about walking out of your house a gazillion times more.

And the same kinds of calculations are the ones that have gone beyond our belief that there's not going to be an issue when the switch is thrown. Could we be wrong? Sure. But if we're going to not do things because we're worried about that, we should not get up in the morning. So if you're willing to get up in the morning and get on the subway, then you should be willing to throw the switch. You should be in fact much more confident about throwing the switch than getting up and taking the subway.

Livio: In fact, way, way, *way* more confident.

Audience: What I really want to know—

Mazur: I think that's another question—but go ahead, next question—

Audience: No, wait, I just want to ask—

Mazur: Next question—

Audience: Can you calculate the chances? You said they're infinitesimal, but how do you know they're infinitesimal? Is there any way of calculating how big they are?

Greene: You make an assumption about how the world works, and within that framework, which you believe from previous experiments, you do a calculation.

Livio: And you calculate, and people have calculated the chances for those black holes forming, and also calculated that even if you form the tiniest of black holes, again, according to those calculations, those are the types of black holes that evaporate in no time, and they will do absolutely nothing. Now, all of that may be wrong, but again, like I said, the chances of me being hit by something when I go out of here are way, way bigger than something happening with that.

Audience: My background is primarily in art, but a little bit in science too, and I had the experience of being required to take calculus in college, coming really strictly out of the arts, and feeling like I was drowning, and every now and then I would get up and break the surface and see the sky, and it was so beautiful, and I'd go right back down and feel like I was drowning again. So I sort of got a glimmer of what beauty could be when you have this kind of discovery of clarity, and the purity of the beauty of it. And I have a son who is a mathematician, who experiences this apparently all the time—Ed knows. He lives in this world.

But this discussion is so great. It reminds me a little bit of the sixties and seventies, when we had a confluence of art and science. There was a real excitement about that, and so it makes me remember that, how art and science can kind of come together and have this very sort of healthy excitement. And I'm not so sure what came out of it, but there's so much more in common between the creative process and this sort of mystical, mathematical process. I just read that, you know, Newton had a mystical experience when he discovered gravity, and depending on what you think the mystical experience is it seems a very strong, good thought to me.

And looking around this room, everyone is here with our brains—we're not really talking about our brains very much, but what if our brains are our premise? It's like our ground. You know, if you learn to fly a plane you actually—you know, it's a big surprise, you get to leave gravity. You actually get to go into a whole other space. So here we are with our brains the way they are, male brains, female brains, you know, with our different structures and predilections, and here we are sort of rising out of our mud of our brains and getting to some nice, clear place for a moment of convergence and everything. And so I think about how beautiful that is and how much it means to me, and I wonder if it means different things to younger people these days. If our sense of beauty has actually evolved, and I think it's possible that it has, which is not to negate any previous sense of beauty, but what if beauty is—you know, my son is very involved in randomness. He's involved in where structure and chaos meets, and I think that he really feels this is beautiful. And so this is another kind of area—I think, being a layperson myself here. I think it's another area. It's kind of an evolution, and some people find the lack of order to be beautiful, and harmony and order—okay.

Levy: How many more questions do we have? Two more? Do we have time for two more?

Mazur: Yes—

Levy: Okay, let's have two more questions.

Livio: I have a train to catch—

Levy: Okay. Very short, because we're running out of tape.

Audience: I'm here as an educator who teaches math, or tries to, aspires to. And I wondered, you know, a lot of us, even if we are not educated well in math we respond to beauty. How as educators we can make the association with beauty in a quicker, more efficient manner, so that more students and more people would respond to beauty as much as they respond to math?

Nersessian: Why don't we have the other question and then you answer both?

Mazur: Okay.

Audience: My name's Jeffrey, I'm a high school science teacher, and my question has to do with my experience in meditation. I find that you can experience anything in the moment as beautiful depending on if you are interpreting what you think on the other thing or if you just experience it directly as it is, without processing that you know what that is. If you're with an unknown mind it feels like everything's more beautiful. And I wondered if when you solve, come up with an elegant solution and you call it beautiful, is it beautiful because you figured out something of how the world works, or is it more to do with that what you thought the universe was you realize it isn't and it becomes more of a mystery? Is that where the beauty comes from?

Nersessian: We have one more question, and you can take all three questions.

Audience: Hi, David, I've done nothing. This is probably irrelevant, but I was wondering about the relationship between certain geometrical figures and their experience by people as being beautiful. I had a playwriting teacher several years ago, and he would tell me that everything, or everything in narrative, takes place in structures of fives, like plays, screenplays, opera, symphonies, marriages [laughter], births—what is it—bullfighting. Basically everything can be mapped into like a geometrical figure that looks like a roller coaster with the high end on the right. So I was just wondering if there was any correlation between the math and the beauty and that. Thanks.

Levy: Thank you. Do you want to take a crack at it—

Livio: I want to say something quickly because I have to leave, in particular about the issue of how do you inspire the younger generation. That's a very big question. I wish it didn't come at the very last moment. But let me say that I think it's the type of discussions that you actually heard here that do the trick, namely, I think that part of the problem is that some of the curricula that are being taught in schools involve too much technical exercising and so on, and they present neither math nor the sciences even as a part of culture. I mean we should somehow convey to our students the fact that just as anybody who finishes high school, let's say, at least wants to read a Shakespeare play, they also need to know what Newton's laws are, and they need to see this bigger picture, where science is a part of the human endeavor to understand the universe. And I think that this can be conveyed via, you know, even more students attending these types of forum, or reading those more interdisciplinary type things and so on, and not just—you know, if you solve twenty more trigonometric problems that's really not going to do it. I mean the ones who will need that trigonometry will probably do enough exercises to use it, but it's the other people that—you know we want young people to go into math and sciences, and even if they don't go into math and sciences, to be part of the people who are somewhat attentive to math and sciences. This is what we want to happen.

Levy: Did you want to say something about—I think in relationship to what you just said, I think that Ed wanted to say a little something about the funding of these particular—

Nersessian: Well, it's because we got some funding from Templeton Foundation that made this roundtable possible.

Levy: And we're very grateful about that—

Nersessian: Barry, do you want to answer—

Mazur: Well, let me add one sentence to—I agree with what Mario says completely, but one thing is to convey the beauty of mathematics to your students, you can't fake it. That is to say, you have to appreciate it, and to appreciate it, if you do it, if you really appreciate it you will convey it.